# $B_{s}^{0} \rightarrow K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ decays within supersymmetry 

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Abstract: We compute the supersymmetric (SUSY) contributions to the observables in $B_{s}^{0} \rightarrow K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ decays. The hadronic parameters in the standard-model (SM) amplitudes are obtained from the $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}$ decay using a recent approach that combines flavor $\mathrm{SU}(3)$ symmetry and a controlled input from QCD factorization. The latest experimental data for $B R\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)$is in agreement with the SM prediction. We study how the branching ratios and the direct and mixing-induced CP asymmetries of both $B_{s}^{0} \rightarrow K \bar{K}$ decay modes are affected with the inclusion of SUSY, after imposing constraints from $B R\left(B \rightarrow X_{s} \gamma\right), B \rightarrow \pi K$ and $\Delta M_{s}$ over the parameter space. While the branching ratios remain unaffected by SUSY, we identify the CP asymmetries of the $B_{s}^{0} \rightarrow K \bar{K}$ decays as the most promising observables to look for large deviations from the SM.

Keywords: B-Physics, CP violation, Supersymmetry Phenomenology.

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## 1. Introduction

Recently, the precision of $B$-physics measurements has increased dramatically due to the experimental results of CDF, Babar and Belle. This implies that new strategies are necessary for controlling hadronic uncertainties. In addition, it is important to identify those observables which are useful for signalling the presence of physics beyond the standard model (SM). Once these are found, the next step is to explore the impact of well-motivated models.

In a recent publication we computed the supersymmetric (SUSY) contributions to $B_{s}^{0} \rightarrow K^{+} K^{-}$decays. In the present paper, we present an update of this analysis, with four important improvements. First, we extend the calculation to include $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ decays. Second, our discussion of the predictions of the SM is based on a new method which uses the $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}$ decay [2]. It combines QCD factorization with flavour symmetries, and represents a substantial improvement in the control of hadronic uncertainties. Third, the limits on the SUSY parameter space include the latest constraints from $B_{s}^{0}-\bar{B}_{s}^{0}$ mixing [3], as well as data from $B \rightarrow X_{s} \gamma$ and $B \rightarrow \pi K$ decays. The fourth point is related to squark mixing. For each fermion, each of the two components (left-handed, right-handed) has a scalar SUSY partner, the squark. One can have mixing of the left-handed or right-handed squarks of the three generations. In ref. [1], it was assumed that one of the two mixings (LL or RR) was zero. In this paper, we allow simultaneous nonzero values of both LL and RR mixing.

We pay particular attention here to the CP-violating asymmetries of the decays $B_{s}^{0} \rightarrow$ $K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$. These are defined as:

$$
\begin{equation*}
A_{\text {dir }}=\frac{|\mathcal{A}|^{2}-|\overline{\mathcal{A}}|^{2}}{|\mathcal{A}|^{2}+|\overline{\mathcal{A}}|^{2}}, \quad A_{\text {mix }}=-2 \frac{\operatorname{Im}\left(e^{-i \phi_{s}} \mathcal{A}^{*} \overline{\mathcal{A}}\right)}{|\mathcal{A}|^{2}+|\overline{\mathcal{A}}|^{2}}, \tag{1.1}
\end{equation*}
$$

where $\mathcal{A}$ is the amplitude of the decay in question, $\overline{\mathcal{A}}$ is the amplitude of the CP-conjugate process, and $\phi_{s}$ is the phase of $B_{s}^{0}-\bar{B}_{s}^{0}$ mixing.

In the $\mathrm{SM}, B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ is dominated by a single penguin decay amplitude. There are other penguin contributions, but they are suppressed and enter only at the level of $\leq 5 \%$. (They are all included in section 2.) The direct CP-violating asymmetry, $A_{\text {dir }}$, involves the interference of the dominant penguin amplitude and the suppressed contributions, and is therefore very small in the SM. As for the mixing-induced CP asymmetry, $A_{\text {mix }}$, since the $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ decay is dominated by a single amplitude, $A_{\text {mix }}$ essentially measures $\phi_{s}$. This phase is very small in the SM [躬, so that this CP asymmetry is expected to be correspondingly small. Since both CP asymmetries of $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ are expected to be so small in the SM, this makes them interesting observables for detecting the presence of new physics (NP). The situation is somewhat different for the decay $B_{s}^{0} \rightarrow K^{+} K^{-}$since it receives a tree contribution which cannot be neglected with respect to the dominant penguin contribution. In the SM, the interference of the penguin and tree amplitudes in $B_{s}^{0} \rightarrow K^{+} K^{-}$gives rise to larger CP asymmetries than in $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$. Both decays involve a $\bar{b} \rightarrow \bar{s}$ transition and therefore have branching ratios of $O\left(10^{-5}\right)$.

All of these SM predictions can change in the presence of SUSY. Naively, one would guess that all SUSY contributions to $B_{s}^{0} \rightarrow K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ are suppressed by $M_{W}^{2} / M_{S U S Y}^{2}$, where $M_{\text {SUSY }} \sim 1 \mathrm{TeV}$, and are therefore small. However, some of the SUSY contributions involve squark-gluino loops, which are proportional to the strong coupling constant $\alpha_{s}$. Compared to the SM, the relative size of these contributions is therefore $\left(\alpha_{s} / \alpha\right)\left(M_{W}^{2} / M_{\text {SUSY }}^{2}\right)$. Since this is $O(1)$, such contributions can compete with those of the SM, leading to significant modifications of the SM predictions for $B_{s}^{0} \rightarrow K^{+} K^{-}$, and especially for $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$. In this paper, we consider only these SUSY contributions, as they are the dominant effects.

Experimentally, the branching ratio of $B_{s}^{0} \rightarrow K^{+} K^{-}$has been measured at CDF (5) :

$$
\begin{equation*}
B R\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)_{\exp }=(24.4 \pm 1.4 \pm 4.6) \times 10^{-6} \tag{1.2}
\end{equation*}
$$

The CP asymmetries for $B_{s}^{0} \rightarrow K^{+} K^{-}$are likely to be measured soon at CDF [6]. However, the measurements for $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ will probably take more time at CDF or may have to wait until LHCb.

In section 2, we discuss the SM expectations for the various observables in $B_{s}^{0} \rightarrow$ $K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$. In order to do this, we must consider certain $B_{d}^{0}$ decays. Here we follow ref. [2] and use $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}$. Section 3 contains the general analysis of $B_{s}^{0} \rightarrow K^{+} K^{-}$ and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ decays with the addition of NP. We discuss the amplitudes for these decays in the presence of NP, as well as strategies for measuring the NP parameters. We turn specifically to SUSY in section 4 and calculate its effect on the amplitudes of $B_{s}^{0} \rightarrow K^{+} K^{-}$ and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$. We note that SUSY can substantially modify the SM predictions for the CP asymmetries while keeping the branching ratios basically unaffected. We conclude in section 5 .

## 2. Standard-model analysis of $B_{s}^{0} \rightarrow K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$

Consider first $B_{s}^{0} \rightarrow K^{+} K^{-}$, which at the quark level is $\bar{b} \rightarrow \bar{s} u \bar{u}$. This decay receives a contribution from a penguin diagram $P E N^{\prime}$ (the prime indicates a $\bar{b} \rightarrow \bar{s}$ transition). The penguin diagram receives contributions from each of the internal quarks $u, c$ and $t$. However, using the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, one can write

$$
\begin{equation*}
P E N^{\prime}=V_{u b}^{*} V_{u s}\left(P_{u}^{\prime}-P_{t}^{\prime}\right)+V_{c b}^{*} V_{c s}\left(P_{c}^{\prime}-P_{t}^{\prime}\right) . \tag{2.1}
\end{equation*}
$$

The decay $B_{s}^{0} \rightarrow K^{+} K^{-}$also receives several other diagrammatic contributions [7]. The most important of these is the tree amplitude, which is proportional to $V_{u b}^{*} V_{u s}: T R E E^{\prime}=$ $V_{u b}^{*} V_{u s} T^{\prime}$. The amplitude can therefore be written ${ }^{1}$

$$
\begin{align*}
\mathcal{A}\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right) & \simeq V_{u b}^{*} V_{u s}\left[T^{\prime}+\left(P_{u}^{\prime}-P_{t}^{\prime}\right)\right]+V_{c b}^{*} V_{c s}\left(P_{c}^{\prime}-P_{t}^{\prime}\right) \\
& \equiv V_{u b}^{*} V_{u s} T^{s \pm}+V_{c b}^{*} V_{c s} P^{s \pm}, \tag{2.2}
\end{align*}
$$

where $P^{s \pm} \equiv\left(P_{c}^{\prime}-P_{t}^{\prime}\right)$ and $T^{s \pm}=\left[T^{\prime}+\left(P_{u}^{\prime}-P_{t}^{\prime}\right)\right]$. Note that $\left|V_{u b}^{*} V_{u s}\right| \simeq 5 \%\left|V_{c b}^{*} V_{c s}\right|$, and this CKM suppression compensates the relative size of the amplitudes $\left|P^{s \pm} / T^{s \pm}\right| \sim 0.1$ (see ref. (2). Thus, the first term is smaller than the second, but must be included in the analysis.

The amplitude for $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ (quark level: $\bar{b} \rightarrow \bar{s} d \bar{d}$ ) can be treated similarly. In this case, there is no tree diagram, but we keep the notation $T^{s 0}$ for the penguin contribution proportional to $V_{u b}^{*} V_{u s}$, leading to

$$
\begin{equation*}
\mathcal{A}\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right) \simeq V_{u b}^{*} V_{u s} T^{s 0}+V_{c b}^{*} V_{c s} P^{s 0} . \tag{2.3}
\end{equation*}
$$

Here, both $T^{s 0}$ and $P^{s 0}$ are of the same size, but since nothing compensates for the strong CKM suppression, the first term is strongly suppressed, leading to a very small direct CP asymmetry.

In the isospin limit, $P^{s \pm}=P^{s 0}$. However, $T^{s \pm} \neq T^{s 0}$ due to the presence of the tree diagram in $B_{s}^{0} \rightarrow K^{+} K^{-}$. Since the terms proportional to $V_{u b}^{*} V_{u s}$ are small, the amplitudes for $B_{s}^{0} \rightarrow K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ are approximately equal in the SM. However, this need not be the case for NP.

In order to determine the amplitudes for $B_{s}^{0} \rightarrow K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$, we need to know $P^{s \pm}, P^{s 0}, T^{s \pm}$ and $T^{s 0}$. These can be obtained by considering $B_{d}^{0}$ decays. One can use $B_{d}^{0} \rightarrow \pi^{+} \pi^{-}$decays in order to determine the hadronic parameters [1], 8- [1]. However, it was noted in ref. [2] that $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}$ provides smaller errors on the predictions, and that the inclusion of some (quite generic) sign predictions from $B_{d}^{0} \rightarrow \pi^{+} \pi^{-}$decays leads to a greater restriction on the SM ranges.

The argument leading to the determination of the $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}$ parameters is as follows. The amplitude for this decay can be written

$$
\begin{equation*}
\mathcal{A}\left(B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}\right) \simeq V_{u b}^{*} V_{u d} T^{d 0}+V_{c b}^{*} V_{c d} P^{d 0} . \tag{2.4}
\end{equation*}
$$

[^0]There are thus three unknown quantities to be determined: the magnitudes of $P^{d 0}$ and $T^{d 0}$, and their relative strong phase. Three pieces of information are therefore needed. One piece of information comes from the measurement of the $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}$ branching ratio: $B R\left(B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}\right)=(0.96 \pm 0.25) \times 10^{-6} 12 .{ }^{2}$

A second piece of information comes from QCD factorization (QCDf) 14, 15. In QCDf the various hadronic quantities can be calculated using a systematic expansion in $1 / m_{b}$. However, a potential problem arises when one encounters endpoint infrared (IR) divergences in higher-order power-suppressed terms. Their evaluation thus requires an arbitrary IR cutoff, and they may be enhanced numerically (for a given IR cutoff). The key observation [2] is that the difference $\Delta_{d} \equiv T^{d 0}-P^{d 0}$ is free of these IR divergences and can be calculated fairly accurately within QCDf:

$$
\begin{equation*}
\Delta_{d}=(1.09 \pm 0.43) \times 10^{-7}+i(-3.02 \pm 0.97) \times 10^{-7} \mathrm{GeV} \tag{2.5}
\end{equation*}
$$

Note that the values of the real and imaginary pieces of $\Delta_{d}$ can be affected by a global phase transformation, so that only the modulus $\left|\Delta_{d}\right|$ is physical. This provides the second constraint on the hadronic parameters of $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}$.

Finally, the authors of ref. [2] find that only values $-0.2 \leq A_{\text {dir }}^{d 0} \leq 0.2$ are consistent with the measured value of $B R\left(B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}\right)$ and the theoretical value of $\Delta_{d}$. This is the third piece of information.

Using the values of the branching ratio and $\left|\Delta_{d}\right|$, as well as the allowed range for $A_{\mathrm{dir}}^{d 0}$, one can obtain the moduli and relative strong phase of the hadronic parameters in $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}:$

$$
\begin{align*}
\left|T^{d 0}\right| & =(1.1 \pm 0.8) \times 10^{-6} \mathrm{GeV} \\
\left|P^{d 0} / T^{d 0}\right| & =1.2 \pm 0.2 \\
\arg \left(P^{d 0} / T^{d 0}\right) & =(-1.6 \pm 6.5)^{\circ} \tag{2.6}
\end{align*}
$$

In fact, there is a twofold discrete ambiguity in determining these quantities, but the authors of ref. [2] argue that only one solution is physical. The argument uses the two methods involving the decays $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}$ and $B_{d}^{0} \rightarrow \pi^{+} \pi^{-}$. It was shown in ref. [2] that the second solution (the unphysical one) requires a large U-spin violation (in the phase) and, moreover, it predicts $A_{\text {dir }}^{s \pm}<0$. This is clearly in contradiction with the prediction for the sign of $A_{\text {dir }}^{s \pm}$ using $B_{d}^{0} \rightarrow \pi^{+} \pi^{-}$, as can be seen in table 1 of ref. [1]. This resolves the two-fold ambiguity.

In addition, it was pointed out that there exists a strong anticorrelation between the signs of $A_{\text {mix }}^{s \pm}$ and $A_{\text {dir }}^{d 0}$, as can be seen in table 1 of ref. [2]. On the other hand, using the method of $B_{d}^{0} \rightarrow \pi^{+} \pi^{-}$one can see that the predicted value for $A_{\text {mix }}^{s \pm}$ is always negative [1]. Putting together both methods, one finds an important restriction on $A_{\text {dir }}^{d 0}$ that prefers positive values. This affects and improves all predictions, shown in table 1 of ref. [2] (only the lower half of this table should be taken).

[^1]Recently, a measurement was reported by the BABAR collaboration [13]:

$$
\begin{equation*}
A_{\operatorname{dir}}\left(B_{d} \rightarrow K_{s}^{0} K_{s}^{0}\right)=-0.40 \pm 0.41 \tag{2.7}
\end{equation*}
$$

This very preliminary measurement, although still quite uncertain, seems to point towards negative values for $A_{\mathrm{dir}}^{d 0}$, but it is compatible with a small positive asymmetry. Note, however, that the measured central value for $A_{\text {mix }}^{d 0}$ is much larger than 1 . Thus, one should really take these numbers only as a proof that they can be measured. Here we present the results for the range $-0.2 \leq A_{\text {dir }}^{d 0} \leq+0.2$ (this includes the non-preferred negative region).

From these, one can now compute the parameters of the $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ amplitude, taking into account $\mathrm{SU}(3)$ breaking [2]. Factorizable $\mathrm{SU}(3)$-breaking corrections are introduced by means of $P^{s 0}=f P^{d 0}$ and $T^{s 0}=f T^{d 0}$, where the factor $f$ is defined as 15

$$
\begin{equation*}
f=\frac{M_{B_{s}^{0}}^{2} F_{0}^{B_{s}^{0} \rightarrow \bar{K}^{0}}(0)}{M_{B_{d}^{0}}^{2} F_{0}^{B_{d}^{0} \rightarrow \bar{K}^{0}}(0)}=0.94 \pm 0.20 . \tag{2.8}
\end{equation*}
$$

and the input values are taken from refs. [6, 15]. This parameter can be calculated on the lattice. Other sources of $\operatorname{SU}(3)$ breaking which are suppressed by $1 / m_{b}$ originate from hard-spectator scattering (differences in the distribution amplitudes of $B_{d}^{0}$ and $B_{s}^{0}$ ) and weak annihilation (diagrams in which the gluon emission comes from the spectator quark). All effects are computed within QCDf (we assume that QCDf gives at least the right order of magnitude), which gives the following bounds [2]:

$$
\begin{gather*}
\left|P^{s 0} /\left(f P^{d 0}\right)-1\right| \leq 3 \%, \\
\left|T^{s 0} /\left(f T^{d 0}\right)-1\right| \leq 3 \% \tag{2.9}
\end{gather*}
$$

Note that large final-state-interaction $\mathrm{SU}(3)$-breaking effects are possible. However, these are common to both $B_{d, s}^{0} \rightarrow K^{0} \bar{K}^{0}$ decays and, consequently, they cancel in relating the two modes. Thus the parameters of the amplitude for $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ can be established (with errors), and the branching ratio and CP asymmetries determined. These numbers are given below.

The decay $B_{s}^{0} \rightarrow K^{+} K^{-}$is somewhat more complicated, as a combination of U-spin and isospin is required to connect $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}$ to $B_{s}^{0} \rightarrow K^{+} K^{-}$. The relations between those hadronic parameters are given in ref. [2], including SU(3)-breaking corrections evaluated within QCDf. The bounds are

$$
\begin{align*}
\left|P^{s \pm} /\left(f P^{d 0}\right)-1\right| & \leq 2 \%, \\
\left|T^{s \pm} /\left(A_{K K}^{s} \bar{\alpha}_{1}\right)-1-T^{d 0} /\left(A_{K K}^{d} \bar{\alpha}_{1}\right)\right| & \leq 4 \%, \tag{2.10}
\end{align*}
$$

where $A_{K K}^{s}$ and $\bar{\alpha}_{1}$ are additional hadronic parameters that can be estimated in QCDf. However, it is also noted that since the $T^{s \pm}$ term is CKM suppressed, any uncertainties in the determination of $A_{K K}^{s}$ and $\bar{\alpha}_{1}$ affect the branching ratio and CP asymmetries of $B_{s}^{0} \rightarrow K^{+} K^{-}$only marginally.


Figure 1: SM predictions for the CP asymmetries in $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ (up) and $B_{s}^{0} \rightarrow K^{+} K^{-}$ (down) as a function of $A_{\text {dir }}\left(B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}\right)$. As explained in the text, the preferred range is the non-shadowed half of the plots $\left[A_{\text {dir }}\left(B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}\right) \geq 0\right]$.

Putting all this together, the SM predictions for the branching ratios and CP asymmetries in $B_{s}^{0} \rightarrow K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ can be obtained. If one conservatively takes all values of $A_{\text {dir }}^{d 0}$ between -0.2 and 0.2 , the prediction for the branching ratio is (2]:

$$
\begin{align*}
B R\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right) & =(18 \pm 7 \pm 4 \pm 2) \times 10^{-6} \\
B R\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right) & =(20 \pm 8 \pm 4 \pm 2) \times 10^{-6} \tag{2.11}
\end{align*}
$$

But if only values of $A_{\text {dir }}^{d 0} \geq 0$ (up to 0.2 ) are taken, the prediction becomes:

$$
\begin{align*}
B R\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right) & =(18 \pm 7 \pm 4 \pm 2) \times 10^{-6} \\
B R\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right) & =(17 \pm 6 \pm 3 \pm 2) \times 10^{-6} \tag{2.12}
\end{align*}
$$

The first error reflects the uncertainty in the QCDf estimates of $\Delta_{d}$ and $\bar{\alpha}_{1}$, as well as in $B R\left(B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}\right)$ (this is the largest uncertainty). The second error corresponds to the uncertainty in $f(\mathrm{SU}(3)$ breaking). The third error introduces a rough estimate of non-enhanced $1 / m_{b}$-suppressed contributions.

The predicted ranges for the CP asymmetries of $B_{s}^{0} \rightarrow K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ within the SM [2] are illustrated in figure 1]. The conservative predictions are [2]:

$$
\begin{align*}
-0.011 & \leq A_{\mathrm{dir}}^{s 0} \leq 0.011 \\
-0.015 & \leq A_{\operatorname{mix}}^{s 0} \leq 0.005 \\
-0.22 & \leq A_{\mathrm{dir}}^{s \pm} \leq 0.49 \\
-0.55 & \leq A_{\operatorname{mix}}^{s \pm} \leq 0.40 \tag{2.13}
\end{align*}
$$

Or, considering only positive values of $A_{\text {dir }}^{d 0}$ :

$$
\begin{align*}
-0.011 & \leq A_{\text {dir }}^{s 0} \leq 0.003, \\
-0.015 & \leq A_{\text {mix }}^{s 0} \leq 0.005, \\
-0.22 & \leq A_{\text {dir }}^{s \pm} \leq 0.49, \\
-0.55 & \leq A_{\text {mix }}^{s \pm} \leq 0.02 . \tag{2.14}
\end{align*}
$$

As expected, the CP asymmetries for $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ are predicted to be very small in the SM, with the prediction that $\left|A_{\text {dir }}^{s 0}\right|$ should be equal or less than $1 \%$ and $\left|A_{\text {mix }}^{s 0}\right|$ should be less than $2 \%$.

## 3. New physics

At present, there are many measurements of $B$ decays and several discrepancies with the SM have appeared. For example, the CP asymmetry in $\bar{b} \rightarrow \bar{s} q \bar{q}$ modes $(q=u, d, s)$ is found to differ from that in $\bar{b} \rightarrow \bar{c} c \bar{s}$ decays by $2.6 \sigma$ (they are expected to be approximately equal in the SM) [16, 17]. In addition, some $B \rightarrow \pi K$ measurements disagree with SM expectations [18], although the so-called $B \rightarrow \pi K$ puzzle [1] has been reduced [19, 20]. One also sees a discrepancy with the SM in triple-product asymmetries in $B \rightarrow \phi K^{*}$ (21, 22], and in the polarization measurements of $B \rightarrow \phi K^{*}$ [23-25] and $B \rightarrow \rho K^{*}$ [26, 27]. Although these discrepancies are not yet statistically significant, there is a unifying similarity: they all point to new physics in $\bar{b} \rightarrow \bar{s}$ transitions. We will therefore follow this indication and assume that the NP appears in $\bar{b} \rightarrow \bar{s}$ decays but does not affect $\bar{b} \rightarrow \bar{d}$ decays. That is, $B_{s}^{0} \rightarrow K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ can be influenced by the NP, but $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}$ [28] is not.

There are many NP operators which can contribute to a given $B$ decay. However, in ref. [29], it was observed that the matrix elements of NP operators carrying non-negligible strong phases are necessarily suppressed with respect to the SM contribution. (Note that each NP contribution can in principle have a different strong phase.) As a result, the relevant NP matrix elements can be combined, and a given $B$ decay thus receives a single NP contribution.

The amplitude for the decay $B_{s}^{0} \rightarrow K^{+} K^{-}$can therefore be written

$$
\begin{equation*}
\mathcal{A}\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)=\mathcal{A}_{S M}^{s \pm}+\mathcal{A}^{u} e^{i \Phi_{u}} \tag{3.1}
\end{equation*}
$$

In the above, $\mathcal{A}_{S M}^{s \pm}$ contains both weak and strong phases, but we have separated out the weak phase $\left(\Phi_{u}\right)$ of the NP contribution. (Note: the NP strong phase is zero, so that $\mathcal{A}^{u}$ is real.) The name of the NP parameters reflects the fact that this decay is $\bar{b} \rightarrow \bar{s} u \bar{u}$ at the quark level.

Similarly, the amplitude for $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ (quark level: $\bar{b} \rightarrow \bar{s} d \bar{d}$ ) is

$$
\begin{equation*}
\mathcal{A}\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right)=\mathcal{A}_{S M}^{s 0}+\mathcal{A}^{d} e^{i \Phi_{d}} . \tag{3.2}
\end{equation*}
$$

If the NP conserves isospin, we have $\mathcal{A}^{u}=\mathcal{A}^{d}$ and $\Phi_{u}=\Phi_{d}$, but in general this need not be the case.

One can make experimental measurements of $B_{s}^{0} \rightarrow K^{+} K^{-}$, obtaining the branching ratio and the direct and mixing-induced CP asymmetries. Assuming that $\mathcal{A}_{S M}^{s \pm}$ is known from $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}$ decays (as in section 2), these three measurements allow one to extract $\mathcal{A}^{u}$ and $\Phi_{u}$, as well as the relative strong phase between the SM and NP amplitudes. One can proceed similarly for $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$. In this way one can measure all the NP parameters [9].

In the following section, we compute the SUSY contributions to $\mathcal{A}^{u}, \Phi_{u}, \mathcal{A}^{d}$ and $\Phi_{d}$, and the resultant branching ratios and CP asymmetries for $B_{s}^{0} \rightarrow K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$.

## 4. SUSY predictions for $B_{s}^{0} \rightarrow K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$

As mentioned above, the relevant SUSY contributions to $\bar{b} \rightarrow \bar{s} q \bar{q}$ transitions come from squark-gluino box and penguin diagrams. We follow the procedure outlined in ref. [1], which is based on the work of Grossman, Neubert and Kagan [30]. We refer the reader to these references for further details.

As shown in ref. [31], the most natural solution to the $B \rightarrow \pi K$ puzzle is the introduction of isospin-breaking NP amplitudes. The isospin-breaking effect is more naturally realized in the present scenario, which allows large up-down squark mass splittings, than in the more popular mass insertion (MI) approximation. Indeed, since we are working in a scenario with near-maximal mixing between bottom and strange squarks, the squark MI approximation is not adequate. Note that the dangerous $\bar{s} \rightarrow \bar{d}$ and $\bar{b} \rightarrow \bar{d}$ flavour-changing neutral currents are not generated in this scenario due to the assumption of vanishing (1,2) and $(1,3)$ components in the scalar down-type squark mass matrix [30].

In this scenario, the SUSY contributions to the Wilson coefficients depend on the following parameters: the masses of the squarks and gluino, two mixing angles $\theta_{L, R}$ and two weak phases $\delta_{L, R}$. These angles parametrize the rotation matrices that diagonalize the left- and right-handed squark mass matrices. The expressions for the NP amplitudes $\mathcal{A}^{u} e^{\Phi_{u}}$ and $\mathcal{A}^{d} e^{\Phi_{d}}$ in terms of these parameters are obtained in complete analogy with ref. [1]. To be specific, we write the expressions for these amplitudes in terms of the Wilson coefficients (denoted by $\bar{c}_{i}^{q}$ and $\bar{C}_{8 g}^{\text {eff }}$ ):

$$
\begin{align*}
\mathcal{A}^{u} e^{i \Phi_{u}} & =\left\langle K^{+} K^{-}\right| \mathcal{H}_{\mathrm{eff}}^{\mathrm{NP}}\left|B_{s}^{0}\right\rangle \\
& =\frac{G_{F}}{\sqrt{2}}\left[-\chi\left(\frac{1}{3} \bar{c}_{1}^{u}+\bar{c}_{2}^{u}\right)-\frac{1}{3}\left(\bar{c}_{3}^{u}-\bar{c}_{6}^{u}\right)-\left(\bar{c}_{4}^{u}-\bar{c}_{5}^{u}\right)-\lambda_{t} \frac{2 \alpha_{s}}{3 \pi} \bar{C}_{8 g}^{\mathrm{eff}}\left(1+\frac{\chi}{3}\right)\right] A \\
\mathcal{A}^{d} e^{i \Phi_{d}} & =\left\langle K^{0} \bar{K}^{0}\right| \mathcal{H}_{\mathrm{eff}}^{\mathrm{NP}}\left|B_{s}^{0}\right\rangle \\
& =\frac{G_{F}}{\sqrt{2}}\left[-\chi\left(\frac{1}{3} \bar{c}_{1}^{d}+\bar{c}_{2}^{d}\right)-\frac{1}{3}\left(\bar{c}_{3}^{d}-\bar{c}_{6}^{d}\right)-\left(\bar{c}_{4}^{d}-\bar{c}_{5}^{d}\right)-\lambda_{t} \frac{2 \alpha_{s}}{3 \pi} \bar{C}_{8 g}^{\mathrm{eff}}\left(1+\frac{\chi}{3}\right)\right] A \tag{4.1}
\end{align*}
$$

where $\chi \simeq 1.18$ and

$$
\begin{equation*}
A=\left\langle\bar{K}^{0}\right|(\bar{b} d)_{V+A}\left|B_{s}^{0}\right\rangle\left\langle K^{0}\right|(\bar{d} s)_{V+A}|0\rangle=i\left(m_{B}^{2}-m_{K}^{2}\right) f_{K} F^{B_{s} \rightarrow K} \simeq 1.42 \mathrm{GeV}^{3} \tag{4.2}
\end{equation*}
$$

For the numerical values we take [4, [5] $F^{B_{s} \rightarrow K}=0.31$ and $f_{K}=0.1598 \mathrm{GeV}$. The explicit expressions for the Wilson coefficients $\bar{c}_{i}^{d, u}$ (which includes both L and R mixing
contributions) can be found in the appendix of ref. [1] ], where some small typos in ref. 30] were corrected.

In order to obtain the NP amplitudes, we must evaluate the Wilson coefficients within SUSY. We consider the following values and ranges for the SUSY parameters:

- $m_{\tilde{u}_{L}}=m_{\tilde{d}_{L, R}}=m_{\tilde{b}_{L, R}}=m_{\tilde{g}}=250 \mathrm{GeV}$
- $250 \mathrm{GeV}<m_{\tilde{u}_{R}}, m_{\tilde{s}_{R, L}}<1000 \mathrm{GeV}$
- $-\pi<\delta_{L, R}<\pi$
- $-\pi / 4<\theta_{L, R}<\pi / 4$

The SM inputs are the same as those used in ref. [2]. The Wilson coefficients are sensitive to the $\tilde{s}-\tilde{b}$ mass splitting. They vanish for $m_{\tilde{s}}=m_{\tilde{b}}$ and grow when the splitting is large. We therefore expect these contributions to be most important for large values of $m_{\tilde{s}}$ (keeping $m_{\tilde{b}}$ fixed). In the same way, NP effects in $\bar{b} \rightarrow \bar{d} q \bar{q}$ transitions depend on the difference $m_{\tilde{d}}-m_{\tilde{b}}$. By setting $m_{\tilde{d}}=m_{\tilde{b}}$ we ensure that $\bar{b} \rightarrow \bar{d}$ decays get no such contributions, which is consistent with the discussion in section 3. A difference between $\mathcal{A}^{u} e^{\Phi_{u}}$ and $\mathcal{A}^{d} e^{\Phi_{d}}$ is only possible in the presence of a nonzero $\tilde{u}-\tilde{d}$ mass splitting. Without it there are no contributions to isospin-violating operators. However, this mass splitting must be very small in the left-handed sector due to $\operatorname{SU}(2)_{L}$ invariance. We therefore set $m_{\tilde{u}_{L}}=m_{\tilde{d}_{L}}$, but allow for a significant mass splitting in the right-handed sector.

There are also constraints on the SUSY parameter space from other processes that have been already measured. The constraints from the decays $B \rightarrow \pi K$ and $B \rightarrow X_{s} \gamma$ are described in ref. [1]. In particular, we take $B R\left(B \rightarrow X_{s} \gamma\right)=(3.55 \pm 0.26) \times 10^{-4}$ [16], with an increased error to cover the various theoretical uncertainties. ${ }^{3}$ Most importantly, one must take into account the recent measurement of $\Delta M_{s}$, which was not included in the previous analysis. The latest measurement [3], together with the SM fit [33, 35], gives ${ }^{4}$

$$
\begin{equation*}
\left(\frac{\Delta M_{s}}{\Delta M_{s}^{S \mathrm{M}}}\right)_{\exp }=(0.81 \pm 0.19) \mathrm{ps}^{-1} \tag{4.3}
\end{equation*}
$$

The constraints from all these measurements have been included in our analysis. Other traditionally-important constraints like $B_{s}^{0} \rightarrow \mu^{+} \mu^{-}$are very sensitive to other SUSY parameters, mostly $\tan \beta$ and $m_{A}$. However, for small values of $\tan \beta$ and values for $m_{A}$ above 200 GeV , they have no effect on our allowed region to SUSY (figure (2).

Taking into account the various constraints, the contributions from LL and RR mixing have been analyzed. $\Delta M_{s}$ is the strongest constraint, and it is the relevant one when considering only LL or RR mixing separately. In particular, it has a large impact on the phases $\Phi_{u}$ and $\Phi_{d}$. In the case of $B_{s}^{0} \rightarrow K^{+} K^{-}$, LL mixing gives the largest contribution

[^2]

Figure 2: SUSY contribution to the NP amplitudes $\mathcal{A}^{u} e^{i \Phi_{u}}$ (left) and $\mathcal{A}^{d} e^{i \Phi_{d}}$ (right) in the scenario with simultaneous LL and RR mixings. The dark regions correspond to the variation of the SUSY parameters over the considered parameter space. The light regions satisfy the experimental bounds, including the recent measurement of $\Delta M_{s}$.
to the amplitude, more than twice that of RR. However, in the case of $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ both contributions are similar in size.

When both LL and RR mixings are allowed simultaneously, the constraints on the SUSY parameter space are changed. In this case new operators for $B_{s}^{0}-\bar{B}_{s}^{0}$ mixing are generated, so that the effect is not simply the combination of the two separate contributions (for instance, see ref. [34]). We find that (i) now $B R\left(B \rightarrow X_{s} \gamma\right)$ is also important, not only $\Delta M_{s}$, and (ii) the global effect of the constraints is weaker. The upshot is that there is a certain enhancement of the NP amplitudes when both LL and RR mixings are combined. In particular, the weak phases $\Phi_{u}$ and $\Phi_{d}$ are not so strongly constrained as when either LL or RR mixing is taken to vanish.

In figure 2 we show the allowed ranges for $\mathcal{A}^{u} e^{i \Phi_{u}}$ and $\mathcal{A}^{d} e^{i \Phi_{d}}$ in the scenario with simultaneous LL and RR mixings. The dark regions correspond to the values that these amplitudes take when varying the parameter space over the initial ranges. The light regions show how these values are reduced by the existing experimental constraints mentioned above. There are two important remarks. First, we see that the above constraints do indeed greatly reduce the allowed SUSY parameter space. Second, even so, the effect on $\mathcal{A}^{u} e^{i \Phi_{u}}$ and $\mathcal{A}^{d} e^{i \Phi_{d}}$ can be significant.

At this stage we can identify what are the effects of the various constraints in reducing the SUSY parameter space. The bound from $B \rightarrow X_{s} \gamma$ affects only the lefthanded sector. In particular, for large $m_{\tilde{s}}$, the regions with $\left|\theta_{L}\right| \gtrsim 10^{\circ},\left|\delta_{L}\right| \lesssim 60^{\circ}$ and $\left|\theta_{L}\right| \gtrsim 10^{\circ},\left|\delta_{L}-\pi\right| \lesssim 60^{\circ}$ are excluded. The bound from $\Delta M_{s}$ is much stronger: when $m_{\tilde{s}} \gtrsim 400 \mathrm{GeV}$, any values of $\left|\theta_{L, R}\right| \gtrsim 5^{\circ}$ are excluded, as well as those regions in which
$\delta_{L}+\delta_{R} \approx-3 \pi / 2,-\pi / 2, \pi / 2,3 \pi / 2 .{ }^{5}$ After these bounds are imposed on the parameter space, the constraints from $B \rightarrow \pi K$ have very little effect on the regions in figure 2 .

Note that the allowed region for $\mathcal{A}^{u}$ is much larger than that for $\mathcal{A}^{d}$, by approximately a factor of 3 . In the isospin limit, these should be equal, so this factor of 3 is a measure of isospin breaking in this NP scenario. In particular, for $m_{\tilde{u}_{R}}=250 \mathrm{GeV}$ (zero $\tilde{u}_{R}-\tilde{d}_{R}$ mass splitting), the values of $\mathcal{A}^{u}$ reduce to those for $\mathcal{A}^{d}$.

We now examine the effect of these contributions on the observables. By adding the SUSY contributions to the SM amplitudes as in eqs. (3.1) and (3.2), it is possible to compute the branching ratio and the CP asymmetries in the presence of SUSY. Figure 3 shows the allowed values for the $B_{s}^{0} \rightarrow K^{+} K^{-}$observables, for three different values of $A_{\mathrm{dir}}^{d 0}$, compared with the predictions of the SM and with the recent experimental value for the $B_{s}^{0} \rightarrow K^{+} K^{-}$branching ratio reported by the CDF collaboration [3] [eq. (1.2)]. The agreement between the CDF measurement and the prediction of the SM in ref. [2] erases any discrepancy between experiment and the SM. This branching ratio will now be an important future constraint. The branching ratio within SUSY should not deviate much from the SM prediction so as not to generate any disagreement with data. Indeed, figure 3 shows that the impact of SUSY on the branching ratio of $B_{s}^{0} \rightarrow K^{+} K^{-}$is practically negligible. Interestingly, for positive values of $A_{\text {dir }}^{d 0}$ (preferred region), the SM predicts a smaller value for $B R\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)$, but it is now compatible with the new data. Still, it is in this case that SUSY shows a larger deviation in the correct direction.

A completely different picture arises for the CP asymmetries. The results for the direct CP asymmetry reveal that SUSY can have an impact. This is not surprising: SUSY introduces a term in the total amplitude which is of the same order of magnitude as that of the SM and carries a weak phase that is not constrained. The mixing-induced CP asymmetry gets affected in a more dramatic way. The interpretation is that the SUSY contribution to the mixing angle $\phi_{s}$ can be large (in fact it can take all values between $-\pi$ and $\pi$ ), while in the SM it is tiny: $\phi_{s}^{S M} \simeq-2^{\circ}$. Any experimental measurement falling inside the dark area in the plots, but outside the dashed rectangle, would not only signal NP but clearly could be accommodated by supersymmetry.

Figure $\square$ shows the results for $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$. Although the branching ratio is little changed in the presence of SUSY, the enhancement of the CP asymmetries due to the inclusion of the SUSY contributions is in this case even more important. The reason is that, within the SM, the CP asymmetries are much smaller in $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ than they are for $B_{s}^{0} \rightarrow K^{+} K^{-}$, because of the absence of the tree diagram. Thus the impact of SUSY is much greater. This is evident by looking at the three lower plots in figure 4 where the tiny rectangles corresponding to the SM predictions can hardly be observed. We have drawn a circle around them to indicate their position.

These are a good illustration of the general scenario discussed in the introduction. While the branching ratios in this case are relatively insensitive to supersymmetry, the direct and mixing-induced CP asymmetries of these decays are greatly affected. Thus, these CP asymmetries are the observables to focus on in order to observe NP, particularly

[^3]

Figure 3: Predictions, in the form of scatter plots, for the correlations between $B R\left(B_{s}^{0} \rightarrow\right.$ $\left.K^{+} K^{-}\right)-A_{\mathrm{dir}}\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)(u p)$ and $A_{\text {mix }}\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)-A_{\mathrm{dir}}\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)$(down) in the presence of SUSY, for a) $A_{\mathrm{dir}}^{d 0}=-0.1$, (b) $A_{\mathrm{dir}}^{d 0}=0$ and (c) $A_{\mathrm{dir}}^{d 0}=0.1$. The dashed rectangles correspond to the SM predictions. The horizontal band shows the experimental value for $B R\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)$at $1 \sigma$.

SUSY, while the branching ratio of $B_{s}^{0} \rightarrow K^{+} K^{-}$can become an important constraint on models beyond the SM other than SUSY.

## 5. Conclusions

In this paper we have considered the branching ratios and CP asymmetries for the decays $B_{s}^{0} \rightarrow K^{+} K^{-}$and $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ in a supersymmetric (SUSY) model, focusing on the dominant gluino-squark contributions [30]. This analysis is an extension and update of ref. [1] , which now includes the SUSY analysis of both $B_{s}^{0}$ decay modes and allows for both LL and RR mixing. In addition, the determination of the SM contributions from the decay $B_{d}^{0} \rightarrow \pi^{+} \pi^{-}$used in ref. [1] has been replaced by the recently-proposed combination of $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}$ and QCD factorization of ref. [2]. This allow us to obtain more precise predictions.

We have included the constraints coming from $B R\left(B \rightarrow X_{s} \gamma\right), B \rightarrow \pi K$ and $\Delta M_{s}$, and we find the following results.

- The new-physics (NP) amplitudes are $\mathcal{A}^{u} e^{i \Phi_{u}}\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)$and $\mathcal{A}^{d} e^{i \Phi_{d}}\left(B_{s}^{0} \rightarrow\right.$ $K^{0} \bar{K}^{0}$ ). We find that both can get significant contributions from SUSY. In the isospin limit, these quantities are equal. However, our calculations show that, for the region of parameters considered, in SUSY there can be a difference of up to a factor of 3 between the NP amplitudes. This indicates the possible level of isospin


Figure 4: Predictions, in the form of scatter plots, for the correlations between $B R\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right)-$ $A_{\text {dir }}\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right.$ ) (up) and $A_{\text {mix }}\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right)-A_{\text {dir }}\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right)$ (down) in the presence of SUSY, for (a) $A_{\mathrm{dir}}^{d 0}=-0.1$, (b) $A_{\mathrm{dir}}^{d 0}=0$ and (c) $A_{\mathrm{dir}}^{d 0}=0.1$. The dashed rectangles correspond to the SM predictions. These are quite small in the three lower plots, so they are indicated by a circle.
breaking in this type of theory. In particular, in the SUSY model considered here, large isospin violation is possible when there is large mass splitting in $\widetilde{u}_{R}-\widetilde{d}_{R}$.

- The branching ratio $B R\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)$is very little affected by SUSY. At most, the SM prediction can be increased by $15 \%$ for $A_{\text {dir }}^{d 0}=0.1$. In fact, SUSY can somewhat improve the already good agreement between the SM prediction and the new precise CDF measurement [5]. The impact of SUSY on $B R\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right)$ is even smaller, reflecting the reduced allowed region for $\mathcal{A}^{d} e^{i \Phi_{d}}$ as compared to $\mathcal{A}^{u} e^{i \Phi_{u}}$.
- The situation is very different for the CP asymmetries; the size of the effect depends strongly on the decay and the type of asymmetry. For $B_{s}^{0} \rightarrow K^{+} K^{-}$, the direct CP asymmetry within SUSY is in the range $-0.1 \lesssim A_{\mathrm{dir}}\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)^{\text {SUSY }} \lesssim 0.7$ for $-0.1 \leq A_{\mathrm{dir}}^{d 0} \leq 0.1$. Depending on the value of $A_{\mathrm{dir}}^{d 0}$, it may be possible to disentangle the SUSY contribution from that of the SM. This is due to the competition between the tree and the NP amplitudes for each value of $A_{\mathrm{dir}}^{d 0}$. As for $A_{\text {mix }}\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)$, its value can vary all the way from -1 to +1 , signaling a large impact from SUSY.
- Turning to $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$, the CP asymmetries are particularly promising. This decay is dominated by the penguin amplitude in the SM, and so the direct CP asymmetry is strongly suppressed: it is predicted to be at most of the order of $1 \%$. However, in the presence of SUSY, the direct CP asymmetry can be 10 times larger. The mixing-induced CP asymmetry is also predicted to be very small in the SM. However,
$A_{\text {mix }}\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right)^{\text {SUSY }}$ covers the entire range, and so this asymmetry can be large in the presence of SUSY.


## Acknowledgments

We thank G. Punzi for helpful discussions. This work was financially supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD) No. KRF-2005-070-C00030 (SB), NSERC of Canada (DL), and by FPA2005-02211 (JM \& JV), PNL2005-41 (JM \& JV) and the Ramon y Cajal Program of Spain (JM).

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[^0]:    ${ }^{1}$ We use the following notation (see ref. 同): quantities carrying the superscripts $d 0, s 0$ and $s \pm$ correspond to $B_{d}^{0} \rightarrow K^{0} \bar{K}^{0}, B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ and $B_{s}^{0} \rightarrow K^{+} K^{-}$, respectively.

[^1]:    ${ }^{2}$ While completing this work, this measurement has been updated by BABAR in ref. 13 to $B R\left(B_{d}^{0} \rightarrow\right.$ $\left.K^{0} \bar{K}^{0}\right)=(1.08 \pm 0.30) \times 10^{-6}$. However, in this work we prefer to stick to the quoted value more near to the HFAG value from ICHEP06

[^2]:    ${ }^{3}$ Interestingly, the latest NNLO calculations in the SM show that $B R\left(B \rightarrow X_{s} \gamma\right)$ (SM) is a little lower than the experimental average 32.
    ${ }^{4}$ Note that here we take the largest fit result for the SM prediction 33] because it falls within the second prediction of ref. [35]. Moreover, the value for $\Delta M_{s}^{S M}$ taken here differs from that used for the bound in ref. (1] that was based on ref. 36].

[^3]:    ${ }^{5}$ For further details of the $\Delta M_{s}$ constraint on this parameter space, see ref. 34.

